The Economics of Space 433: Lecture 21

Spatial Inequality

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Roadmap

- We have studied the role of space in the allocation of economic activity
- A new set of evidence from all over the world across and within countries stresses out the role of space for inequality
- We will study two facets of that
  - Inequality across space
  - How this can change over time

Roadmap

- **Spatial Inequality**
- Spatial Inequality Over Time
- A Model of Spatial Inequality

Measures of Inequality

- There are various inequality measures we could use
- They are all based on looking at the income distribution and computing some statistic
  - For example, the ratio of the 90th to the 10th percentile, or the 90th to the median
  - A very popular one is the Gini coefficient computed using all incomes

$$G = \frac{\sum_i \sum_j |x_i - x_j|}{2N \sum_j x_j}$$

    where x_i is the income of individual i and N is the number of individuals
    - Notice that we normalize G by total income so it is about inequality not growth

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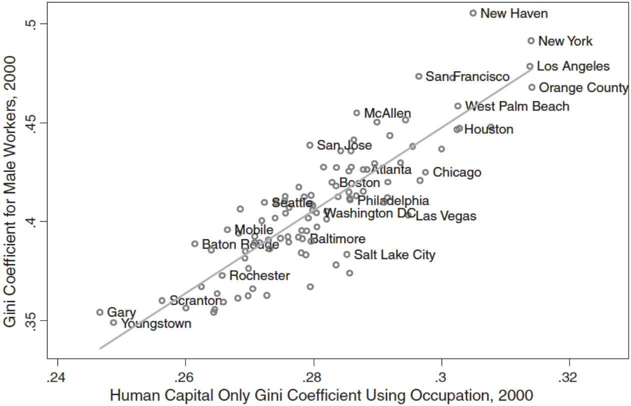
  where x_i is the income of individual i and N is the number of individuals
    - Notice that we normalize G by total income so it is about inequality not growth
- How do you interpret the Gini coefficient?

Gini: Interpretation

- If everyone has the same income, Gini is 0 (lowest inequality).
  - Equality of incomes means that $|x_i - x_j| = 0$ for all i and j, so Gini's numerator is zero.

- If 1% makes 50% of all income the coefficient is at least 24.5%.
  - See Appendix for details and other examples

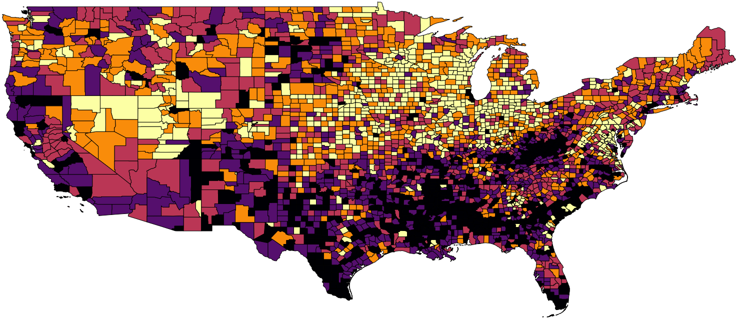
Gini Coefficient by City and Human Capital



Source: Gini coefficients are calculated using the 5 percent Integrated Public Use Microdata Series (IPUMS) for 2000, at usa.ipums.org.

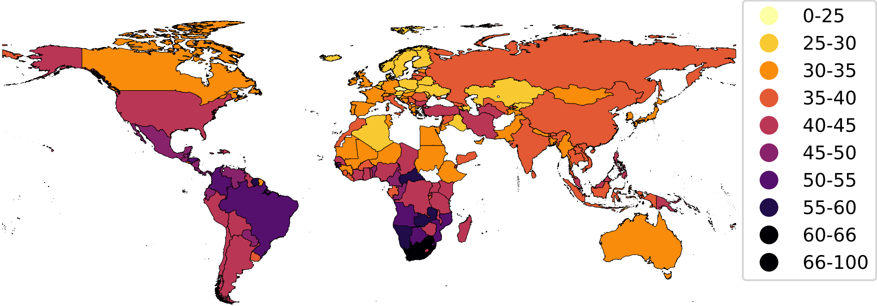
Gini coefficients for 2000. Note: Human capital computed as years of schooling. Source: Glaeser et al. '09

Gini Coefficient by County



Gini coefficient by county, 2015. Data plotted by decile, with darker colors representing higher Gini.
Source: American Fact Finder (Census)

Gini Coefficient by Country



Gini coefficient by country, most recently reported as of 2019.
Source: World Bank

Roadmap

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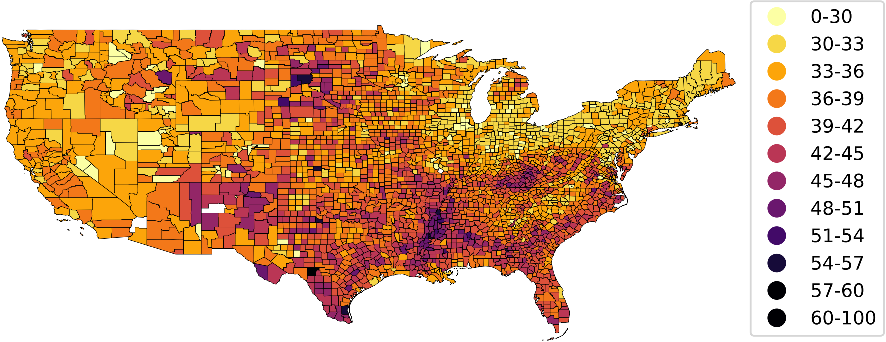
Inequality in Space and Time

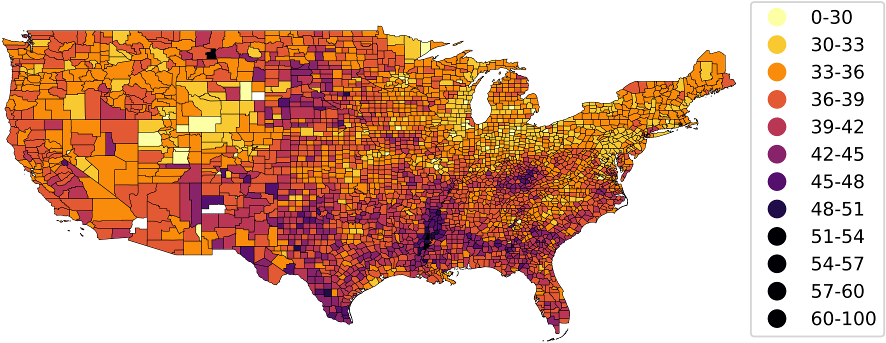
- How does inequality in space change across time?
- Substantial changes when looking across and within countries
  - United States is a *characteristic example* of increase in inequality over the past 3 decades

Gini Coefficient by County over time, 1970



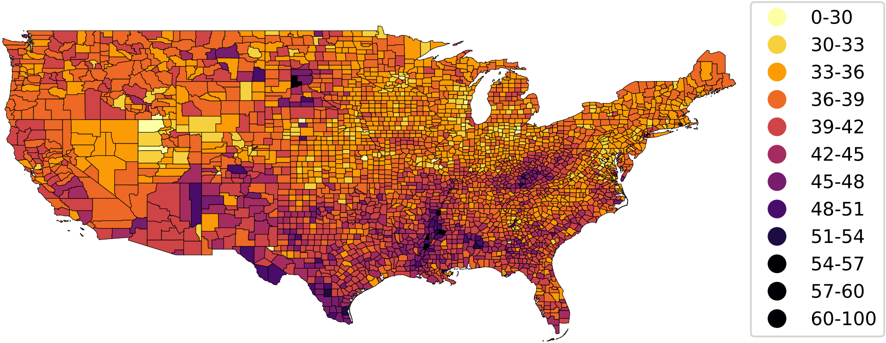
Gini coefficient by county, 1970. Data plotted by in levels.
Source: https://nielsen.sites.oasis.unc.edu/data/data.html

Gini Coefficient by County over time, 1980



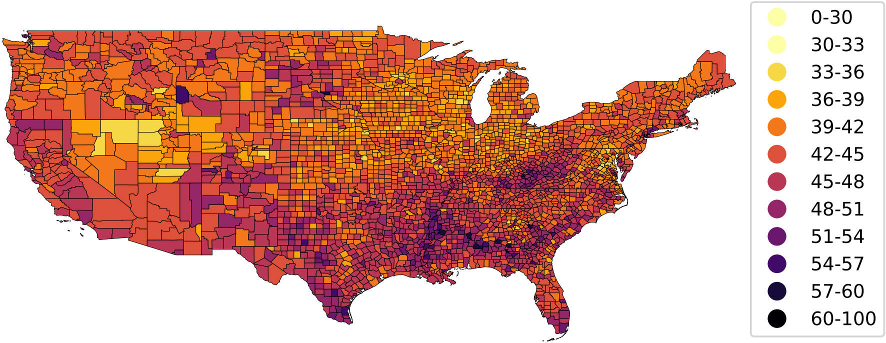
Gini coefficient by county, 1980. Data plotted by in levels.
Source: https://nielsen.sites.oasis.unc.edu/data/data.html

Gini Coefficient by County over time, 1990



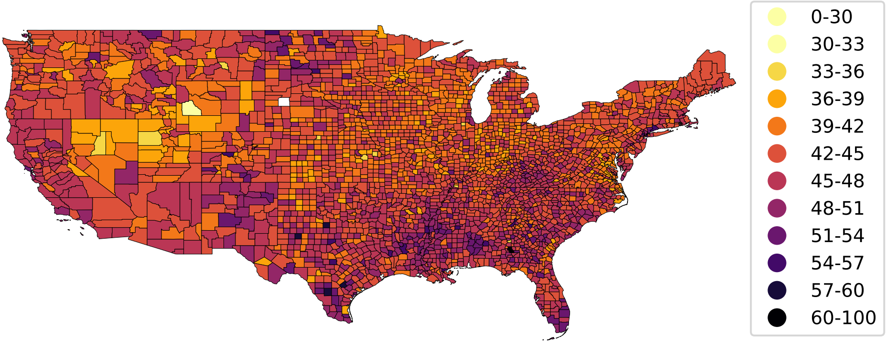
Gini coefficient by county, 1990. Data plotted by in levels.
Source: https://nielsen.sites.oasis.unc.edu/data/data.html

Gini Coefficient by County over time, 2000



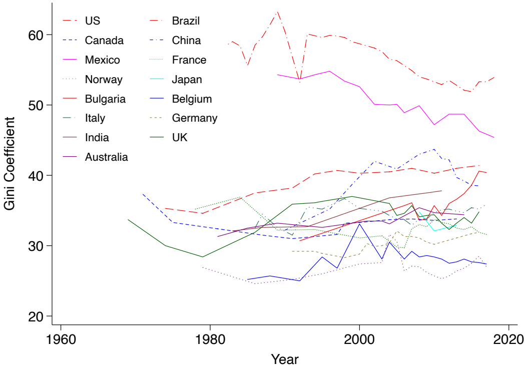
Gini coefficient by county, 2000. Data plotted by in levels.
Source: American Fact Finder (Census)

Gini Coefficient by County over time, 2015



Gini coefficient by county, 2015. Data plotted by in levels.
Source: American Fact Finder (Census)

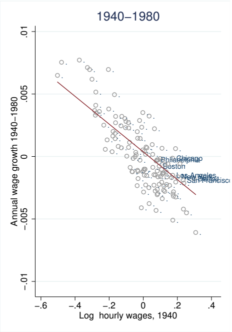
Gini Coefficient by Country Since World War II



Gini coefficient Across Countries since World War II.
Source: World Bank

Spatial Inequality: Convergence or Divergence?

Figure 1: Wage Convergence across Cities before and after 1980

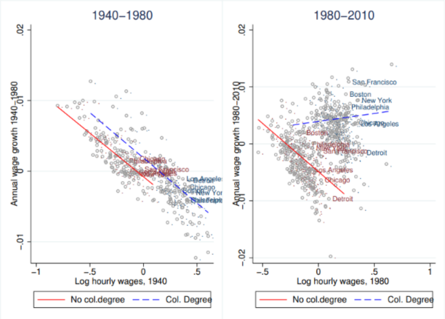


Note: This figure plots each city's (demeaned) annual average wage growth against its (demeaned) initial wage level. The left side depicts 1940-1980; the right side depicts 1980-2010. The size of each city's circle is proportionate to its initial population. The red line depicts a weighted least square bi-variate regression. Data come from the 2010 US Census and the 2010 American Community Survey.

Hourly wage growth by MSA. Source: Giannone '21

Spatial Inequality: Convergence or Divergence?

Figure 2: Wage Convergence across MSAs before and after 1980 by Skill Group



Note: This figure plots each MSA's annual average wage growth (demeaned) against its (demeaned) initial wage level by skill type (highly skilled and less skilled workers). The left depicts 1940-1980; the right depicts 1980-2010. Each MSA's circle size is proportionate to its initial population size by skill group. The red solid and the blue dashed line in each graph depict a weighted least square bi-variate regression, respectively, for less and highly skilled workers.

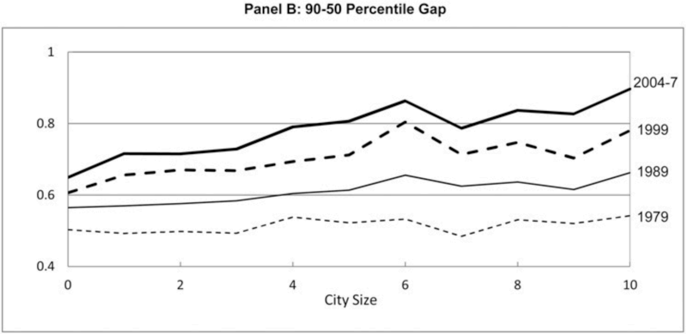
Hourly wage growth by MSA and skill level

Source: Giannone '21

Spatial Inequality Over Time

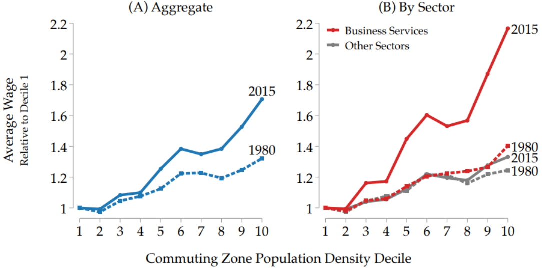
- Inequality has increased in almost every single county in the United States
- But across the world it rises in some countries and declines in others
- We will dive deeper into the US landscape of inequality

City Size and Inequality over Time



Wage Gap by City Size. Source: Baum-Snow, Pavan '13. Public micro data

Skill Premium Growth Across Sectors



Notes: This figure shows average wages across commuting zones (Tolbert and Sizer, 1996) sorted into deciles of increasing population density. Each decile accounts for one-tenth of the US population in 1980. The average commuting zone in decile 1 has a population density of 10 $people/mi^2$ and in decile 10 of 2300 $people/mi^2$. The underlying data come from the US Census Bureau's Longitudinal Business Database. We compute average wages as average payroll per worker by aggregating establishment payroll numbers and employment counts across all establishments in a commuting zone and sector.

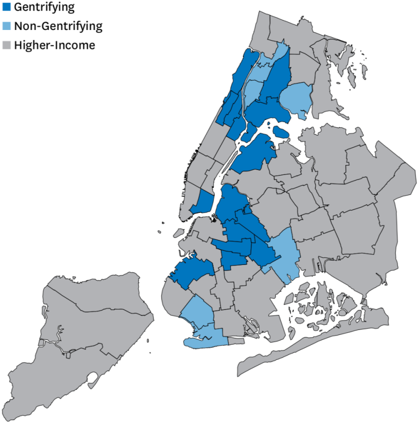
Skill premium growth and commuting zone size. Source: Eckert '19, Eckert Ganapati Walsh '25. US Census public micro data

Spatial Inequality Over Time: Gentrification

- Gentrification is the process of renovating deteriorated neighborhoods
  - Sometimes it is almost synonymous to affluent people moving into poor neighborhoods
- Major North American cities have seen the dramatic effects of gentrification
  - On city structure, house prices, crime rates, household relocation

New York City Gentrification

Figure 1: Classification of Sub-Borough Areas

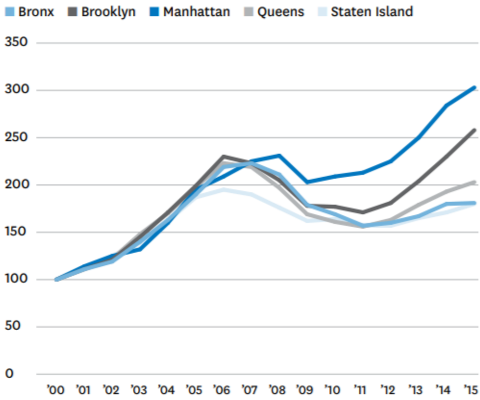


Source: NYU Furman Center

Source: NYU Furman Center, State of NYC Housing 2015, courtesy Sun Kyoung Lee

New York City Gentrification

Figure 3: Index of Housing Price Appreciation for All Residential Property Types (Except Cooperatives) by Borough (Index = 100 in 2000)



Sources: New York City Department of Finance, NYU Furman Center

Source: NYU Furman Center, State of NYC Housing 2015, courtesy Sun Kyoung Lee

Roadmap

- Spatial Inequality
- Spatial Inequality Over Time
- **A Model of Spatial Inequality**

A Model of Spatial Inequality

- How do we model spatial inequality?
- Very robust finding in the literature: people with different skills are paid differently
  - E.g. College vs High School educated
  - Any theory needs to be able to address that robust regularity

A Model of Spatial Inequality

- How do we model spatial inequality?
- Very robust finding in the literature: people with different skills are paid differently
  - E.g. College vs High School educated
  - Any theory needs to be able to address that robust regularity
- This requires a distinct role for different types of labor in the production function
  - We assume firm production function is

$$Y_i = \bar{A}\left(\left(\bar{A}_i^H L_i^H\right)^{\frac{\rho-1}{\rho}} + \left(\bar{A}_i{}^L L_i^L\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}$$

  - Define L_i^H, L_i^L high- and low- skill labor and i is the location
    - e.g. supply of college educated and high school educated in location i
  - $\bar{A}_i^H, \bar{A}_i^L$ are relative importance of labor of type H, L
  - ρ elasticity of substitution of production between the two types of labor
  - If $\rho \to \infty$ and $\bar{A}_i^H = \bar{A}_i^L$, we go back to $Y_i = \bar{A}\left(L_i^H + L_i^L\right)$ i.e. only total labor matters

Firm Cost Minimization

- How does firm allocate labor?
- Solves a cost minimization

$$\min_{L_i^H, L_i^L} w_i^H L_i^H + w_i^L L_i^L$$

$$\text{s.t. } \left(\left(\bar{A}^H L_i^H\right)^{\frac{\rho-1}{\rho}} + \left(\bar{A}^L L_i^L\right)^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} = 1$$

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- Idea of minimization: firm needs to hire the right proportions of workers when attempting to produce one unit of the output

Firm Optimal Hiring

- How does it allocate labor? Solves a cost minimization

$$\min_{L_i^H, L_i^L} w_i^H L_i^H + w_i^L L_i^L$$

$$\text{s.t.} \quad \left(\left(A_i^H L_i^H \right)^{\frac{\rho-1}{\rho}} + \left(A_i^L L_i^L \right)^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} = 1$$

- First order conditions with respect to the two choices

$$w_i^H = \lambda \left(A_i^H \right)^{\frac{\rho-1}{\rho}} \left(L_i^H \right)^{\frac{-1}{\rho}}, \; w_i^L = \lambda \left(A_i^L \right)^{\frac{\rho-1}{\rho}} \left(L_i^L \right)^{\frac{-1}{\rho}}$$

which give the following 'skill premium'

$$\frac{w_i^H}{w_i^L} = \left(\frac{A_i^H}{A_i^L} \right)^{\frac{\rho-1}{\rho}} \left(\frac{L_i^H}{L_i^L} \right)^{-1/\rho}$$

  - Obviously: higher relative supply means lower relative wages

Firm Optimal Hiring

- Optimal choice of skills

$$\frac{w_i^H}{w_i^L} = \left(\frac{A_i^H}{A_i^L}\right)^{\frac{\rho-1}{\rho}} \left(\frac{L_i^H}{L_i^L}\right)^{-1/\rho}$$

- How many high skill and low skill people will sort in each area?
  - Welfare equalization will hold for each skill level

$$W^H = \frac{w_i^H}{P_i} u_i^H, \quad W^L = \frac{w_i^L}{P_i} u_i^L \quad \text{for all } i$$

Firm Optimal Hiring

- Optimal choice of skills

$$\frac{w_i^H}{w_i^L} = \left(\frac{A_i^H}{A_i^L}\right)^{\frac{\rho-1}{\rho}} \left(\frac{L_i^H}{L_i^L}\right)^{-1/\rho}$$

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- I intentionally added superscript-indices on u_i's, u_i^H, u_i^L.
  - Do high- and low-skill people have the same tastes?
  - Arguably not. Diamond '16 provides evidence that amenities that attracted more high-skill people improved much faster in cities in US during 1980-2000
    - Retail, transportation infrastructure, crime, environmental, schools.
  - This can result in even larger inequality on ''well-being'' between two skill groups than suggested by the differences in skill-premium

Skill-Biased Technical Change

- If $\rho > 1$, improvements in technology of high-skill to low skills change inequality in the corresponding direction
  - In particular increases in $\frac{A_i^H}{A_i^L}$, increases inequality in region i
- Economists argue that recent technological improvements such as computers etc. are complementary to the high-skill people
  - i.e. increase A^H faster than they increase A^L for each i
  - Attribute part of the increase in inequality in skill-biased technical change
- Another interesting possibility is that $A_i^H = \bar{A}_i^H \left(L_i^H\right)^{\alpha}, A_i^L = \bar{A}_i^L \left(L_i^H\right)^{\alpha}$ so that

$$\frac{A_i^H}{A_i^L} = \frac{\bar{A}_i^H}{\bar{A}_i^L} \left(\frac{L_i^H}{L_i^L}\right)^{\alpha}$$

i.e. skill premium increases (under conditions) with the share of high skill workers as in Giannone's findings or in relation to gentrification

Using Big Data to Understand Spatial Inequality

- This is the era of big data
  - We can do much better than looking at samples of cities for some people
  - We can look at the universe of people at a fine geographic detail (e.g neighborhood)

- Datasets such as the Danish micro data offered these detail for years
  - Chetty and his coauthors linked various datasets (e.g. tax etc.) to achieve the same level of detail for the USA post 1940

References

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- The Determinants and Welfare Implications of US Workers' Diverging Location Choices by Skill: 1980-2000, Diamond, 2016. American Economic Review, 106(3), pp. 479-524.
- Inequality and City Size, Baum-Snow, Pavan, 2013. Review of Economics and Statistics, 95(5), pp. 1535-1548.
- Skill-Biased Technical Change and Regional Convergence, Giannone, 2021.
- Growing Apart: Tradable Services and the Fragmentation of the U.S. Economy, Eckert, 2019
- Urban-Biased Growth, Eckert,Ganapati, Conor Walsh, 2025

Appendix: Derivations for Gini Coefficient

- If 1% makes 50% of all income the coefficient is at least 24.5%.
  - Proof uses the Lorenz curve.
  - (Lorenz curve is a graphical representation of the income/wealth distribution).

- If only 1 person (say, the first individual, $n = 1$) makes all income, then $G = \frac{(N-1)x}{Nx}$.
  - Gini's denominator is: $2N \sum_j x_j = 2Nx_1 = 2Nx$.
  - Numerator is:

$$\sum_j |x_1 - x_j| + \sum_{i \neq 1} \sum_j |x_i - x_j| = (N-1)|x_1 - 0| + \sum_{i \neq 1} |x_i - x_1| =$$

$$= (N-1)x_1 + (N-1)|0 - x_1| = 2(N-1)x_1 = 2(N-1)x$$

  - If N is a large number, Gini is close to one: $\lim_{N \to \infty} G = \lim_{N \to \infty}(1 - \frac{1}{N}) = 1$.