

# The Slug Algorithm

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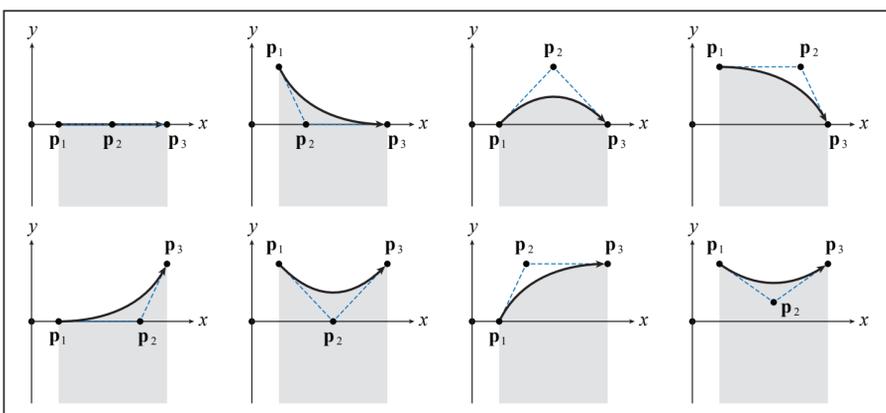
## Root Eligibility

Class	$y_3 < 0$	$y_2 < 0$	$y_1 < 0$	Root 2	Root 1
A	0	0	0	0	0
B	0	0	1	1 -	0
C	0	1	0	1 -	1 +
D	0	1	1	1 -	0
E	1	0	0	0	1 +
F	1	0	1	1 -	1 +
G	1	1	0	0	1 +
H	1	1	1	0	0
				0x2E	0x74

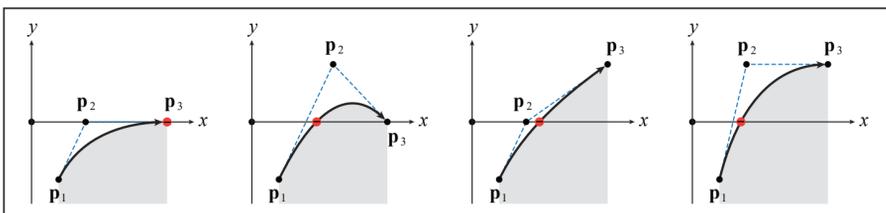
## Winding Number

Quadratic Bézier curve	$\mathbf{p}(t) = (1-t)^2 \mathbf{p}_1 + 2t(1-t) \mathbf{p}_2 + t^2 \mathbf{p}_3$	$\mathbf{p}_i = (x_i, y_i)$
Ray intersection equation	$p_y(t) = (y_1 - 2y_2 + y_3)t^2 - 2(y_1 - y_2)t + y_1 = 0$	
Potential solutions	$t_1 = \frac{b - \sqrt{b^2 - ac}}{a}$ $t_2 = \frac{b + \sqrt{b^2 - ac}}{a}$ $\frac{d}{dt} p_y(t_1) \leq 0$ $\frac{d}{dt} p_y(t_2) \geq 0$ $a = y_1 - 2y_2 + y_3$ $b = y_1 - y_2$ $c = y_1$	
Change to winding number for ray in positive x direction	$+ \text{sat}(k p_x(t_1) + \frac{1}{2})$ if root 1 eligible <span style="color: green;">+</span> $- \text{sat}(k p_x(t_2) + \frac{1}{2})$ if root 2 eligible <span style="color: red;">-</span>	$k = \text{pixels per em}$
Change to winding number for ray in negative x direction	$- \text{sat}(\frac{1}{2} - k p_x(t_1))$ if root 1 eligible <span style="color: red;">-</span> $+ \text{sat}(\frac{1}{2} - k p_x(t_2))$ if root 2 eligible <span style="color: green;">+</span>	

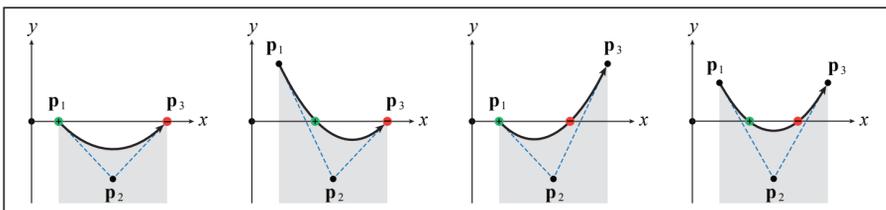
## Class A



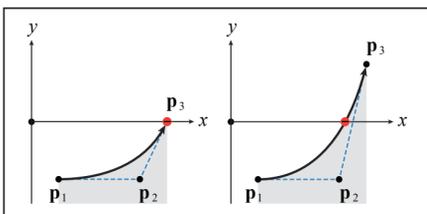
## Class B



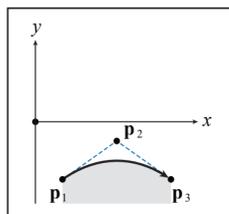
## Class C



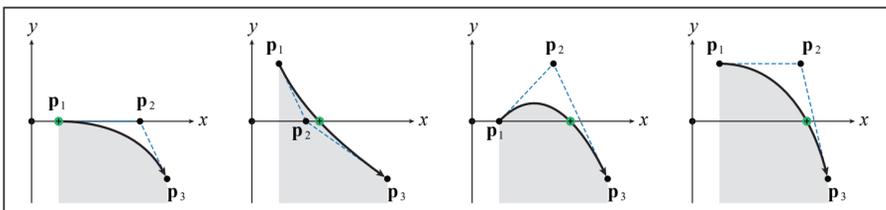
## Class D



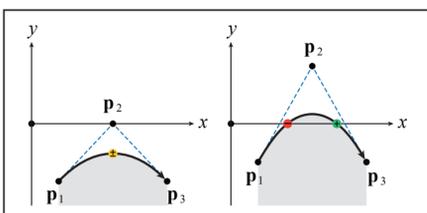
## Class H



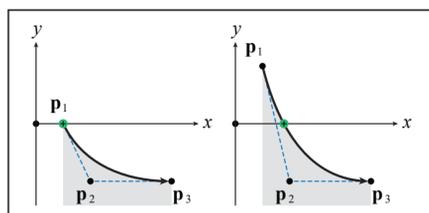
## Class E



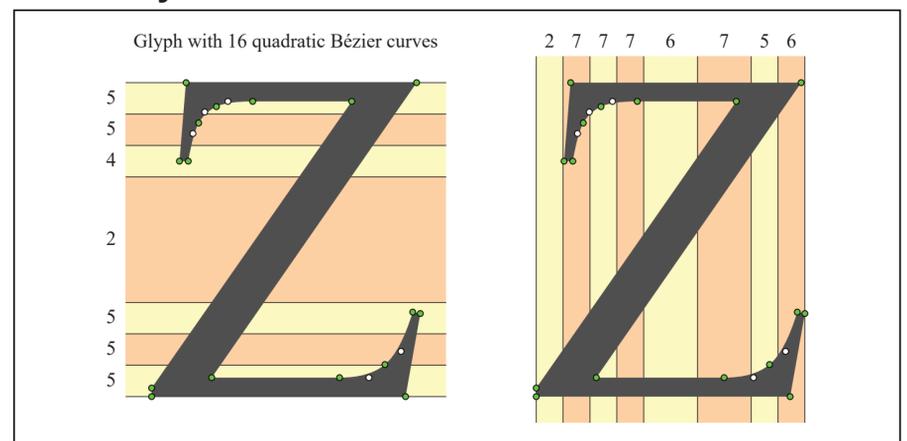
## Class F



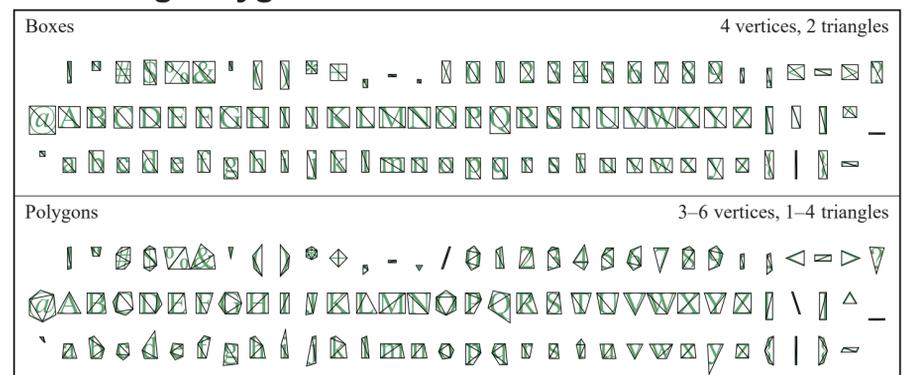
## Class G



## Geometry Bands



## Bounding Polygons



## Dynamic Dilation

$\mathbf{m}$  = transformation matrix from object space to clip space  
 $w$  = viewport width     $h$  = viewport height  
 $d$  = object-space dilation distance  
 $(p_x, p_y)$  = object-space vertex position  
 $(n_x, n_y)$  = object-space vertex normal     $\hat{\mathbf{n}} = \mathbf{n} / \|\mathbf{n}\|$

$$\Delta x = \frac{w}{2} \left[ \frac{m_{00}(p_x + d\hat{n}_x) + m_{01}(p_y + d\hat{n}_y) + m_{03}}{m_{30}(p_x + d\hat{n}_x) + m_{31}(p_y + d\hat{n}_y) + m_{33}} - \frac{m_{00}p_x + m_{01}p_y + m_{03}}{m_{30}p_x + m_{31}p_y + m_{33}} \right]$$

$$\Delta y = \frac{h}{2} \left[ \frac{m_{10}(p_x + d\hat{n}_x) + m_{11}(p_y + d\hat{n}_y) + m_{13}}{m_{30}(p_x + d\hat{n}_x) + m_{31}(p_y + d\hat{n}_y) + m_{33}} - \frac{m_{10}p_x + m_{11}p_y + m_{13}}{m_{30}p_x + m_{31}p_y + m_{33}} \right]$$

$(\Delta x)^2 + (\Delta y)^2 = \frac{1}{4}$  (half-pixel viewport-space dilation)

$$s = m_{30}p_x + m_{31}p_y + m_{33} \quad t = m_{30}\hat{n}_x + m_{31}\hat{n}_y$$

$$u = w \left[ s(m_{00}\hat{n}_x + m_{01}\hat{n}_y) - t(m_{00}p_x + m_{01}p_y + m_{03}) \right]$$

$$v = h \left[ s(m_{10}\hat{n}_x + m_{11}\hat{n}_y) - t(m_{10}p_x + m_{11}p_y + m_{13}) \right]$$

$$d = \frac{s^2 t + s^2 \sqrt{u^2 + v^2}}{u^2 + v^2 - s^2 t^2}$$