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Launch of: 'The Radiant Stranger'
A sculpture by James Lunney and David Grouse, celebrating William Rowan Hamilton

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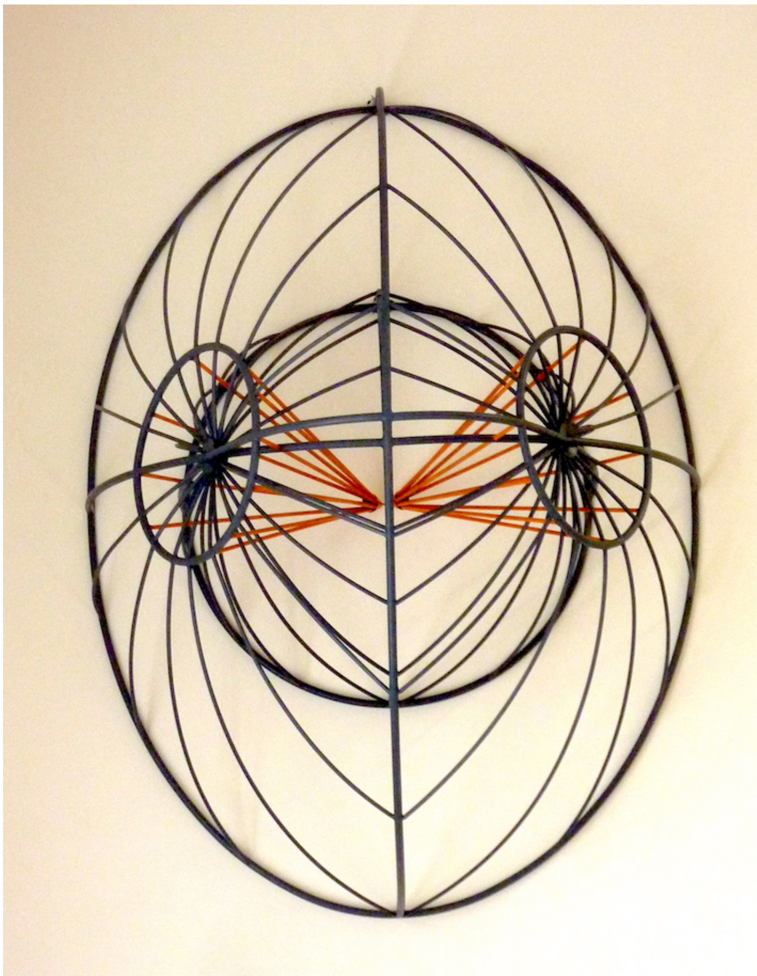
We celebrate this mysterious sculpture, this 'Radiant Stranger', created by James Lunney and David Grouse, and an astonishing story: the discovery, in the early 1830s, by William Rowan Hamilton – then in his mid-twenties – of a physical phenomenon. A 'Stranger'? Its strangeness is obvious.



But why 'Radiant'? Because it concerns light. The sculpture is a shape, abstractly representing light in a crystal. Abstractly, because it represents directions rather than positions. The orange straight lines emerging from the central points are some of the rays travelling in different directions. The lengths of the lines represent the speed with which they travel. The speeds are different because in a crystal the atoms are arranged in a regular array that distinguishes directions. The points where the rays – the orange lines – stop, lie on a complicated surface, here represented by the curved lines that lie in it. It is complicated because actually there are two surfaces, one inside the other, because in such crystals two rays travel in any given direction, and they have different speeds. The class of crystals that Hamilton's surface represents is the most general, where all three directions (forwards, sideways and up) are different.

The surfaces are connected at the two points where the curved lines meet. Near these points, the two surfaces form a double cone, like the old 'diabolo' spinning toys. This double surface is what Hamilton discovered. It is a distillation of the essence of the optics of crystals, completing the subject that others had developed in the preceding decades. You see two conical points. Actually there are four, because the sculpture represents half of the complete structure. The other half would be behind the wall, a perfect reflection of what is in front.

Hamilton was a primarily mathematical man, and we don't consider him as a scientist close to effects that can be observed. But he didn't stop with this abstract shape. He devised a brilliant way in which it could be observed. Take a crystal in this general class, cut into a slab whose surface is a right angles to the ray along the axis of one of the cones, and shine a thin beam of light into it. He predicted that the light would spread into a hollow cone inside the crystal, and emerge as a hollow cylinder; on a screen inserted into its path, the cylinder would appear as a ring of bright light. This is the phenomenon of 'conical refraction'.

Easier said than done, If you get the direction wrong, you see two bright points rather than a ring. But Hamilton's Trinity College colleague, the Reverend Humphrey Lloyd, using a crystal of aragonite, eventually got it right, observing conical refraction and bringing Hamilton's theory to life. He wrote:

"This phenomenon was exceedingly striking. It looked like a small ring of gold viewed upon a dark ground; and the sudden and almost magical change of the appearance, from two luminous points to a perfect luminous ring, contributed not a little to enhance the interest."

Hamilton's work was part of a wider explanatory scheme, in which he treated the paths of light rays and the trajectories of material objects (planets, for example) within the same theoretical framework. This is central to our thinking in physics today: the term 'Hamiltonian', denoting a theoretical structure central to his scheme, appears in almost every scientific paper on theoretical physics. And yet nowadays Hamilton's prediction of conical refraction is virtually unknown to most physicists. Some of us, particularly James Lunney and Denis Weaire here in Dublin, are trying to make our physicist colleagues familiar with Hamilton's seminal discovery, but it is still largely confined to a brief mention near the back pages of a few texts in optics – apart from specialists as I will explain later.

It was different back in the 1830s. Hamilton's discovery was widely appreciated and made him instantly famous. Indeed it created a sensation. here is Airy, then the Astronomer Royal (and not given to praising other people):

"Perhaps the most remarkable prediction that has ever been made";

Here is the senior astronomer Herschel: "... theory actually remanding back experiment to read her lesson anew; informing her of facts so strange, as to appear to her impossible, and showing her all the singularities she would observe in critical cases she never dreamed of trying".

Here is Lloyd himself: "...in the highest degree novel and remarkable... singular and unexpected consequences of the undulatory theory, not only unsupported by any facts hitherto observed, but even opposed to all the analogies derived from experience. If confirmed by experiment, they would furnish new and almost convincing proofs of the truth of that theory." (This was before he carried out his experiment).

Here is Whewell, a leading philosopher-scientist and public intellectual: "In the way of such prophecies, few things have been more remarkable than the prediction, that under particular circumstances a ray of light must be refracted into a conical pencil, deduced from the theory by Professor Hamilton, and afterwards verified experimentally by Professor Lloyd."

The enthusiasm was not restricted to Britain and Ireland. Here is the German mathematician Plücker: "No experiment in physics has made such an impression on my spirit as that of conical refraction. A ray of light entering a crystal and emerging in the shape of a cone of light: something fabulous without precedent…I confess that I thought it impossible to see verified by experiment such an extraordinary result. But since Mr. Lloyd has demonstrated that the experiments are in full concordance with the predictions by Mr. Hamilton, all prejudice against a theory so marvelously confirmed must vanish."

Why such universal astonishment? Because conical refraction was an example of something unfamiliar in the early 1800s: a completely new phenomenon, predicted by purely mathematical reasoning. By the early 1800s, it was widely appreciated that mathematics is essential to understanding the natural world. But the phenomena whose explanation was made possible by mathematics were themselves familiar: tides, eclipses, planetary orbits, machinery.... Nowadays, the prediction of something qualitatively new using theoretical reasoning is commonplace. We have a new theory of something, expressed in equations, and we always ask: what is hidden in this mathematics that we didn't imagine before? It is commonplace today, but in the 1830s it was startling. Only a few years before it had been discovered that a beam of light entering a crystal could split into two ('double refraction'); but splitting into a cone of infinitely many rays was totally unanticipated.

Here's an aspect of Lloyd's experiment that has a double resonance today. He noticed that the bright ring was polarised: the light waves vibrated sideways in a well-defined direction, that rotated by half a turn as he looked at different points around the ring. Today we see this as probably the first example of something much more general, called the 'geometric phase', that occurs in a wide variety of wave phenomena, in particular the quantum physics of a large class of new materials. The second resonance concerns the relations between Hamilton and Lloyd. Lloyd wrote that he hadn't seen this half-turn rotation of the direction of polarisation mentioned in Hamilton's writings, but he was sure Hamilton knew it. This generosity was reciprocated: Hamilton replied that indeed he had known it, but he didn't write it, so Lloyd deserved the priority for discovering it experimentally. A respectful and uncompetitive mutual generosity between colleagues, worth emulating today.

Allow me to complement this account of Hamilton's conical refraction with some personal remarks. Over many years, I had occasionally seen the effect described briefly (in the back of optics texts, as I mentioned), and often thought that one day I ought to seriously understand it. That day came in the early 2000s, when I was involved in the theory of some other aspects of polarised light in crystals. When I looked into the literature, I found that over the nearly two centuries since Hamilton's discovery there were scattered papers drawing attention to different aspects of the phenomenon, and certain refinements: Hamilton's ring was really two close rings, not one, inside the inner ring was a series of faint interference rings, in the middle, a bright spot... But there wasn't a comprehensive theory explicitly connecting all the different features. Based on earlier work, I created this theory. In 2004 I published it.

That same year, I visited Dublin for an unrelated reason giving a J L Synge Memorial lecture, and had the pleasure of meeting our radiant sculptor (one of the pair) James Lunney. He asked if I knew that the following year – 2005 – Dublin would celebrating the bicentenary of Hamilton's birth in 1805. I did not. Then he asked if I had heard of conical refraction, and I replied that coincidentally I had just written a paper about it. James replied: "Would you like to see conical refraction in my laboratory?"

Now there is something that you who are not scientists might not know. We theoretical physicists spend our lives buried in our equations: mysterious squiggles and scribbles that sometimes enter our computers. We are pathetically grateful when anyone shows us, in the world outside our heads, the phenomena we are studying theoretically. Therefore I jumped at the chance to visit James's lab. Within just a few minutes, we were able to identify, in the rings projected onto the screen, every one of the features I had been studying. Extraordinary, gratifying, magical!

Immediately I wanted to know where James had obtained the crystal, hoping I could get one and demonstrate conical refraction in the little laboratory I have at home, where I occasionally play. I was surprised to learn from James, and later confirm by searching the literature, that just as theory papers on conical refraction were few, there were not many experimental studies of the phenomenon. The reason is that it is not easy to grow crystals thick enough for the rings to be big enough to see, and at the same time pure enough for defects not to obscure the effect. He told me that he had bought his from a German company, VCT technologies, who were growing and marketing crystals to demonstrate conical refraction. It was their way of celebrating Hamilton's bicentenary year.

I wanted one! And when I learned where the crystals were grown, I decided that rather than order one online I would to visit the place and collect it in person. Why? Because the crystals were grown under the green fields just outside Hanover, in the old nuclear bomb shelter previously owned by Deutsche Telekom. If there had been a war between NATO and the Warsaw Pact countries, that would have been a command and communications centre. The place was perfect for growing crystals: a constant temperature, and below layers of sand and dust that would filter out the small particles dreaded by crystal growers.

So, I got my crystal: a matchstick of monoclinic double tungstate. It came in a nice case, with a laser and some lenses and polarisers as a demonstration kit. I was apprehensive about taking it through Hanover airport security as part of my hand luggage. And indeed one of the inspectors asked me to open the case. Before I had a chance to explain, she asked: “Ah, teleskopen?” “Yes, of course”, I replied, which seemed to satisfy her.

When I got home, I eagerly went to my lab to test it out, and discovered that the quality was better than the one that James had – actually two, as I discovered later – so that as well as demonstrating conical refraction it would be possible to make detailed measurements to test the theory. In fact, it seems that James’s two and my one were the only three of these crystals that VCT sold, so it was not a very successful venture on their part. But it was very successful for us: with my student Mike Jeffrey, and encouragement and advice from James, we did indeed successfully test the theory, and the three of us published a detailed account of the work.

Science does not stand still. Any discovery is a springboard for more; it raises further questions. Mike Jeffrey and I extended the theory to crystals with a twist – that is, that come in left or right-handed forms – and to crystals that absorb light differently in different directions. Both generalisations radically change the theoretical predictions. And here in Dublin James Lunney, John Donegan, Paul Eastham, together with their students, have pursued the experimental investigations far beyond what we mere playful theorists could.

For several years afterwards, I gave lectures on conical refraction, and delighted in ending by declaring what seemed to be something unique: although conical refraction demonstrates a concept at the very heart of crystal optics, it does not occur naturally anywhere in the universe and it doesn’t have any useful applications. (I did hear about conical refraction being used in gunsights made by Sweden in the first World War, but didn’t take this application seriously.) No longer. When I gave the talk in Paris, and made the same comment at the end, someone – Gabriel Sirat – came to me at the end and said that later that very same afternoon there would be a celebration of the opening of a company (Bioaxial) devoted to using conical refraction in a new kind of high-resolution microscopy. And here in TCD, James and colleagues have been exploring ways to use the conical refraction cylinder to manipulate small particles, adding to the repertoire of what conventional optical tweezers can do. And in Dundee Amin Abdolvand developed a new kind of laser based on conical refraction. Conical refraction optics, both theoretical and experimental, has moved on, and is now a thriving field, which I haven’t followed.

One final thing. Hamilton’s cones have been rediscovered at least twice, in contexts different from the original optics but mathematically and conceptually identical. In quantum chemistry, ‘conical intersections’ between different energy levels of electrons in molecules, fast-moving under the influence of the heavy sluggish nuclei, are now recognised as important in understanding the properties of the molecules. Most chemists are unaware that the idea was ‘prediscovered’ in the early nineteenth century. And in the new classes of materials I briefly mentioned earlier, especially thin films or involving surfaces in a fundamental way, again there are conical intersection of electron energy levels, also, just as in optics, involving directions. But in this field they are called ‘Dirac cones’, because of a formal resemblance between the mathematics and the Dirac equation in relativity. Now, Dirac was among the two or three greatest scientists of the twentieth century and is justly revered, especially in my city – Bristol – where he was born. But Dirac never drew or even mentioned a cone in this context, so naming these new cones after him, rather than Hamilton, is a historical mistake that I am trying to correct, though without much hope: a terminology once established is almost impossible to change, and there is Arnold’s law: no discovery is credited to the right person (this also applies to Arnold’s law itself).

It is fitting that here in Dublin today we can recognise, and with this sculpture celebrate in concrete form, Hamilton’s discovery: for its historical precedence – and indeed prescience – and for its many extensions, descendants, ramifications...