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REVIEW

ALISA BOKULICH
Reexamining the Quantum-Classical Relation: Beyond Reductionism and Pluralism

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Quantum mechanics and classical mechanics are magnificent structures, each with vast explanatory reach. In their usual formulations, the two theories look very different. Classical physics describes the motion of particles in terms of their positions and velocities, influenced by forces acting between them and from outside. In quantum physics, dynamical variables are operators, acting on states in a Hilbert space of vectors, with evolution determined by a Hamiltonian operator. Nevertheless, there is considerable overlap in the phenomena they describe. Although nobody planning an extraterrestrial mission would use Schrödinger's equation as a starting point for programming the trajectories of rockets, few physicists doubt that quantum mechanics applies to planets and spacecraft as well as atoms. The subtle and intricate relations between the classical and quantum worlds are the subject of this very welcome book by Alisa Bokulich.

The quantum-classical connection is a special case of the philosophical problem of theory reduction. As Bokulich explains, conventional approaches to reduction are centred on two contrasting views. In the first ('imperialism' or 'theoretical serial monogamy'), a more general theory, once conceived and validated by experiment or observation, immediately supersedes its less general predecessor theory. Thus, chemistry is regarded as a branch of quantum physics, notwithstanding difficulties in calculating molecular structure and spectra or reaction rates. In the second approach ('isolationism' or 'promiscuous realism'), we live in a 'dappled world', in which each science maintains its separate domain of applicability, with chemistry, biology, geology, etc., retaining their repertoires of concepts and techniques. Bokulich rejects

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this polarization, and most physicists who reflect on their craft would agree with her.

She begins her analysis of the relations between quantum and classical physics by a sensitive and detailed exposition of the views of three quantum pioneers. Werner Heisenberg regarded classical and quantum mechanics as perfect, complete, and separate structures that will never be changed, each describing phenomena in their separate domains of applicability. Nowadays this seems a strangely restricted view; as Bokulich points out, it not only fails to delineate what the 'separate domains of applicability' are, but is also discordant with the manner in which classical concepts played an essential role in Heisenberg's creation of quantum mechanics. Niels Bohr's view was more nuanced. For him, quantum mechanics is a 'rational generalization' of classical mechanics, involving correspondences between the two theories at every level. Usually, Bohr's 'correspondence principle' is interpreted narrowly, as relating frequencies of light involved in transitions between states with large quantum numbers to the frequencies of associated classical trajectories (Bokulich does not emphasize that the quantum numbers must be close as well as large). A more general interpretation of correspondence is that classical and quantum predictions must agree in the limit when Planck's constant h can be neglected. Even this fails to capture what Bohr intended, which is that classical concepts permeate quantum physics not just in the 'classical limit' but at every level. Paul Dirac's view was very different, perhaps reflecting his education in electrical engineering and applied mathematics. For him, both classical and quantum mechanics are 'open theories': approximate descriptions of phenomena, evolving and changing in response to new experiments and theoretical insights.

Of the three approaches, Dirac's is closest to that advocated by Bokulich, but there are two important features ignored by the pioneers and also by almost all philosophers of science with the notable exception of Robert Batterman. The first, that she discusses but in my view does not emphasize enough, is that the limit $h \to 0$ is mathematically singular. This was missed by the pioneers because of their emphasis on the connections between the two theories at the level of formalism (e.g., relating classical Poisson brackets to the commutators of quantum operators), rather than the much richer and more subtle connections between the solutions of the formalism (e.g., wavefunctions and energy levels). Bokulich illustrates the singular limit with the example of particles encountering a potential barrier with sufficient energy to cross it: classically, they slow down but eventually all are transmitted; quantum mechanically some are reflected, the fraction being exponentially small in h and therefore mathematically singular. This is an unnecessarily sophisticated example; the singular limit arises even in the most elementary situation where two equally intense beams of particles overlap. Classically, the intensity is the sum of those in the separate beams ($1 + 1 = 2$). Quantum mechanically, the beams interfere, and the spacing of the fringes separating regions of constructive and destructive interference is proportional to h; across the fringes the intensity varies from zero and four times that of each beam ($1 + 1 \neq 2$) and the classical limit is achieved only after spatial averaging ('decoherence'), reflecting the inability of experiment to resolve fringes on infinitely fine scales. A more far-reaching manifestation of the singularity is that the classical limit $h \to 0$ and the long-time limit $t \to \infty$ cannot be interchanged. This apparently arcane observation resolves much of the confusion about the quantal implications of classical chaos, i.e., persistent instability, which emerges in the long-time limit: confined quantum systems can mimic chaotically evolving classical ones, but ultimately (after times that get larger as h gets smaller) the chaos is suppressed.

The second important previously neglected feature is Bokulich's central and original contribution: classical structures, far from being redundant anachronisms superseded by concepts from quantum mechanics, are playing an important role in explaining quantum phenomena. The explanations are based on mathematical approximations to the full quantum theory that have come to be known collectively as 'semiclassical mechanics'. As vividly expressed in an adaptation of a quotation attributed to Boris Kinber in the related context of the ray limit of wave optics, this is 'sewing the quantum flesh on the classical bones'. Bokulich elaborates this point of view with three case studies, which form the intellectual core of her book.

The first is the ground state of the helium atom: the lowest energy level of two electrons repelling each other and attracted by the nucleus. This played an important historical role in the development of quantum mechanics. When Bohr created what has come to be called 'old quantum theory', he explained the energy levels of the single electron in the hydrogen atom by postulating quantum rules that selected particular classical orbits. This spectacular success stimulated others to try to generalize the rules to determine the energy levels of more complicated systems. Of these attempts, the most far-reaching was Einstein's, but even his generalization could be applied only to classical trajectories that were multiply periodic; in modern terms, non-chaotic trajectories. In particular, nobody succeeded in quantizing helium. This failure led to old quantum theory being regarded as merely a way-station leading to the full quantum theory, within which accurate numerical schemes for solving the Schrödinger equation enabled the ground state of helium to be determined in agreement with experiments. These computations established that quantum mechanics gives a correct description, but did not provide insight into the nature of the quantum state of the electrons. Only in the early 1990s was it realized, by the late Dieter Wintgen and his colleagues, that the failure of early attempts to quantize helium was not a failure of old quantum theory but arose from inability to identify the correct classical structure in this example of the three-body problem (two electrons and the nucleus). When the relevant classical orbits were identified, and incorporated into a modern semiclassical approximation to quantum mechanics, the energy thus calculated agreed closely with the 'exact' energy that had been computed from Schrödinger's equation. This study broke new ground in the classical three-body problem, an example (in the spirit of Dirac's 'open theory' approach to science) of quantum physics leading to deeper understanding of classical physics.

Why bother—why make an approximate calculation when an exact formalism is available? Bokulich offers three reasons, which also apply to her two other examples:

> First, the semiclassical treatments provide an *investigative tool* [...] to investigate physical domains that might not yet be accessible either experimentally or with a fully quantum calculation. Second, they provide a *calculational tool*: semiclassical calculations [...] can be less cumbersome than full quantum calculations. Finally, they provide an *interpretive tool*: [...] physical insight into the structure of a problem, in the way that a fully quantum-mechanical approach might not.

The second case study is the spectrum of high excited energy levels of atoms in strong magnetic fields; these are called Rydberg atoms. Bokulich writes that

> These atoms call to mind Tom Stoppard's play *Hapgood*, in which he writes 'there is a straight ladder from the atom to the grain of sand, and the only real mystery in physics is the missing rung. Below it, [quantum] particle physics; above it, classical physics; but in between, metaphysics' [...]. As an atom that *is* the size of a grain of sand, Rydberg atoms are ideal tools for studying the 'metaphysics' of the relation between classical and quantum mechanics.

In the 1960s, experiment had revealed unexpected resonant structure in the spectrum, associated with the outermost electron, for energy ranges in which this electron was expected to be torn off, leading to an ionized atom with a much simpler spectrum. It took twenty years for the explanation to emerge, and as with helium the concepts were semiclassical. The first step was to understand that motion of the electron is largely chaotic, as the result of conflict between the elliptical 'Kepler' orbits the electron would have under the action of the nucleus alone, without the magnetic field, and the helical paths it would have in the magnetic field alone. The second step was to modify a semiclassical theory that had been developed by Martin Gutzwiller, in which the quantum spectrum—the collective of energy levels—is related to the spectrum of classical periodic orbits, that is, the set of those orbits that repeatedly traverse the same path. One modification was to smooth the spectrum, reflecting the finite resolution of the experiments; this had been anticipated by Roger Balian and Claude Bloch, and led to a representation in terms of a small subset of the infinity of classical orbits. The more fundamental modification, by John Delos and collaborators, was to realize that the force the electron experiences has a singularity at the position of the atomic nucleus, so the relevant orbits are those that start and end at the position of the nucleus: closed, but not periodic. Then a semiclassical calculation, exquisitely blending classical concepts with the quantum notion of wave interference, succeeded in reproducing the full variety of the observed spectrum.

The third case study concerns calculations for ‘quantum billiards’: wavefunctions representing quantum particles confined in planar enclosures where the corresponding classical trajectories would be chaotic. Standard quantum algorithms enable the states to be computed. In 1984, Eric Heller made the important observation that some of the states exhibit regions of high intensity, centred on classical periodic orbits, and gave a semiclassical argument indicating why such ‘scars’ should exist, even though the orbits are all unstable—yet another example of classical reasoning giving insight into a quantum phenomenon.

Bokulich seems to imply that scars conflict with my 1977 prediction that the wavefunctions of highly excited states can be modelled by random functions represented by many interfering waves, based on the semiclassical argument that typical chaotic trajectories pass many times through the same region. In fact there is no conflict, and the reason is an instructive illustration of the subtlety of the semiclassical quantum domain. Heller’s scars are an asymptotic phenomenon, in the sense that the scars get more distinct for high excited states where they are less obscured by the finite quantum wavelength. But they are transitory asymptotic phenomena: although there are always some scarred states, they get rarer higher in the spectrum, and are of measure zero in the extreme asymptotic regime. There, almost all states are not concentrated on periodic orbits; indeed, they explore the enclosure uniformly on average (according to an earlier theorem by Alexander Shnirelman), and randomly on finer scales, both features reflecting the behaviour of the classical billiard motion.

The co-existence of scars and random waves without contradiction provides a fine illustration of another distinction that Bokulich mentions but does not explore, referring instead to discussions of it by Robert Batterman: between ‘universal’ and ‘particular’ phenomena. This entered physics during the 1960s in the attempt to understand how the thermodynamic properties of materials emerged from the statistical mechanics of their microscopic constituents. The particular phenomena are system specific and distinguish one material from another; they include the boiling point of water and the temperature at which iron loses its magnetism. More interesting and fundamental are universal phenomena, such as the manner in which the compressibility becomes infinite as critical points are approached, according to scaling laws that are quantitatively identical for a vast range of materials. The understanding of this universality generated deep and unanticipated insights, including the emergence of fractal structures on all scales, obstructing the smooth reduction of statistical physics to thermodynamics: the many-particle limit is singular, as is the classical limit $h \to 0$. In quantum billiards, scars are particular phenomena, because the individual periodic orbits on which the wave intensity is concentrated depend on the shape of the billiard boundary. The random-wave regime is universal: quantitatively the same for all boundary shapes provided only that the corresponding classical motion is chaotic.

The distinction between the universal and the particular appears not only in the morphology of quantum wavefunctions of classically chaotic systems, but also in the arrangement of their high-lying energy levels. The universal features are the statistics of correlations between close-lying levels, for example, the probability distribution of the spacings between nearest neighbours. In a footnote, Bokulich speculates '[...] that one could cook up a [...] statistical asymptotic agreement between classical and quantum mechanics [...] without having [...] law-like correspondence between the classical and quantum structures.' Exactly this 'statistical asymptotic agreement' is provided by random-matrix theory, a branch of mathematics that is logically independent of quantum theory and has many applications in other areas of science but which reproduces with high accuracy the statistics of energy levels. The explanation of random-matrix universality is an application of Gutzwiller's relation between spectra and periodic orbits. According to this, short-range structure in the energy spectrum is associated with very long classical periodic orbits—a connection that is a consequence of the uncertainty principle relating time and energy. But long classical orbits display a beautiful universality of their own, discovered in the early 1980s by John Hannay and Alfredo Ozorio de Almeida, and it soon became clear that the quantum universality follows from this, though the details are subtle and still being elaborated. An unanticipated implication of the connection between spectra and periodic orbits was that correlations between distant levels are associated with short orbits, and because these are not universal the level correlations will not be universal either; these are the particular phenomena, depending on the details of the classical system, just like wavefunction scars in quantum billiards.

Bokulich's detailed case studies raise an important question, to which she devotes the final chapters. The discovery of quantum mechanics as a deeper theory revealed classical trajectories as temporary structures that dissolve under close scrutiny and can now be discarded. How can nonexistent objects form the basis of explanation and give insight, as semiclassical physics clearly indicates? The same problem arose in optics, where the explanatory structures are the caustics: singularities on which ray families are focused, clearly visible as the brightest features in optical images (e.g., the dancing lines of focused sunlight on the bottoms of swimming pools). In the 1960s, it became clear, through the mathematical discoveries of René Thom and Vladimir Arnold, that caustics are restricted to certain universal forms. It was soon realized that this universality extends to the wave patterns that decorate caustics in the limiting regime of small wavelengths and smooth away their singularities. Some people wondered why we were using properties of nonexistent singularities, when the same phenomena could be 'explained' without them, by numerically solving the fundamental wave equations.

I confess to being puzzled that people find these questions puzzling. The explanatory structures (periodic orbits, ray caustics) are models, constructs that help our understanding. We do not find it problematic that the answer to the question 'Where is my dinner?' is 'On the table', even though a table is our convenient name for a model, assigning significance to what we now know to be an assembly of molecules consisting of atoms whose electrons and nuclei move in largely empty space. Bokulich surveys several philosophical theories of how models can explain, and concludes that none of them can give a satisfactory account of current understanding based on semiclassical analysis as in her three case studies. She ends by sketching her own 'interstructuralism': an attempt to replace the contrasting ideas of the imperialist and isolationist theories with an approach that combines aspects of both, closer to Bohr and Dirac than to Heisenberg. Her conclusions justify the explanatory power of concepts from a superseded theory when used in the theory that replaces it. In brief, 'fictions can explain'.

Bokulich fully appreciates many subtleties that practicing physicists occasionally understand intuitively, but are rarely explicit about. Her ideas are refreshing and original and presented with clarity and erudition. I unreservedly recommend her book to anyone wanting to understand the intricate connections between the classical and quantum worlds.