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The singularities of crystal optics

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This is a summary of our recent paper [1], where we reconstruct the classical theory of crystal optics [2] in a way that emphasises and extends recent ideas of singular optics [3, 4]. We concentrate on the general case where the crystal is chiral, i.e. optically active (gyrotropic) and absorbing (dichroic) as well as biaxially anisotropic, and describe the properties of the crystal as a function of propagation direction **s**. The refractive indices and polarizations of plane waves are eigenvalues and eigenfunctions of the 2x2 part of the reciprocal dielectric tensor □ perpendicular to **s**. For transparent anisotropic crystals, □ is real symmetric; addition of chirality makes □ complex hermitian; addition of dichroism makes □ complex nonhermitian. These different possibilities greatly influence the refractive indices and eigenpolarizations.

Using a new formalism involving projection from the sphere of directions **s** to the stereographic plane $\mathbf{R}=X,Y$, and associated complex variables $Z=X+\mathrm{i}Y$, we obtain convenient explicit formulas for the two refractive indices and polarizations. Particular use is made of a representation of the polarization states as the complex ratios of their vector components.

This enables three types of polarization singularity to classified and explored. First are *singular axes*, which are degeneracies where the two refractive indices are equal. For a transparent non-chiral crystal, these condense pairwise onto the familiar optic axes. In the general case the singular axes are branch points (in **R** space) connecting the two refractive index sheets (figure 1), contrasting with the conical connections characterising optic axes. As absorption is switched on, each optic axis splits into a pair of singular axes, which approach and annihilate as chirality is switched on and increased.

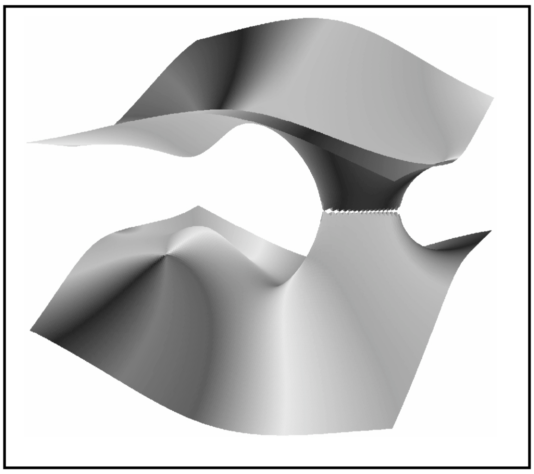

Figure 1. Real part of eigenvalue sheets above the projective plane of directions, for a dichroic gyrotropic birefringent crystal. The sheets are connected across a cut joining the two singular axes, and shaded according to the phase of the ratio of components of the corresponding complex eigenpolarizations in a circular basis.

Second are *C points*, where the polarization is purely circular (right- or left-handed). As well as handedness, each C point has a topological index, whose possible values are +1 (uniaxial tranparent nonchiral crystals), +1/2 (transparent nonchiral, transparent chiral, or dichroic chiral crystals) or +1/4 (an unprecedented situation, corresponding to dichroic nonchiral crystals). In a transparent nonchiral crystal, the C points coincide with the optic axes; as chirality is switched on, the C axes remain fixed while the singular axes migrate away (figure 2) – they are the ghosts of departed singular axes ('haunting theorem').

Third are L lines, where the polarization is purely linear. These divide **R** space into regions with right- and left-handedness, containing the C points (figure 2).

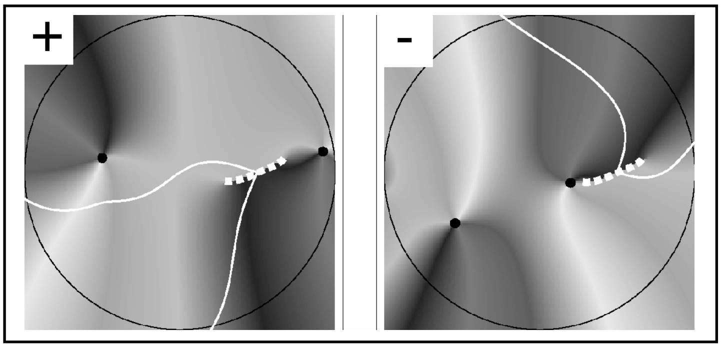

Figure 2. Phases of the ratio of components of the two eigenpolarizations corresponding to figure 1, in the northern hemisphere of directions (within the black circles), showing the singular axes (ends of the cut indicated as white dashed lines), the C points (phase singularities, indicated as black dots) and the L lines (white).

With an explicit local model, it is possible to capture essential features of the general theory, and illustrate this with interference figures generated by slabs of crystal viewed directly or through a polarizer and/or analyzer, enabling the more unfamiliar singularities to be displayed directly (figure 3).

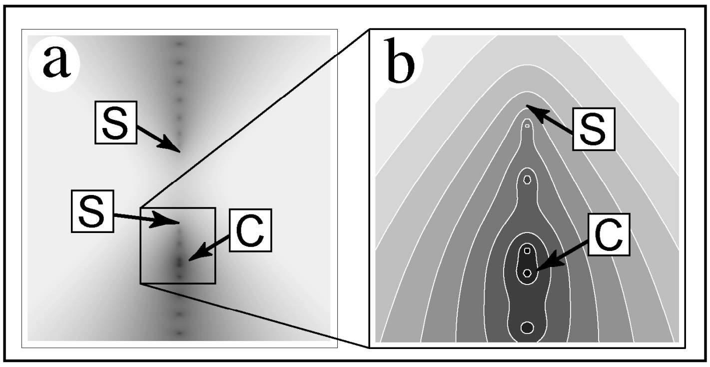

Figure 3. Density plots of intensity for dichroic chiral crystal viewed between crossed circular polarizer and analyzer, near a pair of singular axes. (a) Destructive interference gives two lines of dark spots, ending near the singular axes (labelled S); the additional dark spot labelled C is a C point. (b) Magnification of (a), including contours of intensity, showing more clearly the C point, and the location of the singular axis.

Full details can be found in [1].

[1] M.V.Berry and M.R.Dennis, *The optical singularities of birefringent dichroic chiral crystals* Proc. Roy. Soc. A., in press (2003).

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[3] J.F.Nye, *Natural focusing and fine structure of light: Caustics and wave dislocations* (Institute of Physics Publishing, Bristol, 1999).

[4] M.S.Soskin and M.V.Vasnetsov, *Singular Optics* Progress in Optics, **42**, 219-276 (2001).