

For book

Global properties of simple quantum systems – Berry's phase and others

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Foreword (published in Chinese) by

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Many different areas of physics have been found to be related by the ideas of anholonomy, phase, adiabaticity and parallel transport. It is the emergence of these connections that has surprised me most in the thirteen years since I wrote the first paper [1] on the subject, and it is precisely this aspect that Professor Li strives to emphasize in this book. It is not paradoxical to assert that since 1984 the history of the geometric phase has stretched further and further into the past, as earlier papers involving similar ideas have come to light. I have written elsewhere about these anticipations of the phase [2, 3], and so choose instead to indulge here in some more local history, and describe the way I came to write my 1984 paper.

An interesting and unexplored corner of theoretical physics that emerged in the late 1970s was the quantum mechanics associated with classical chaos. We realized that a key role is played by the statistics of the quantum energy levels. When there is classical chaos, the levels repel each other, and in particular the probability that neighbouring levels have spacing S vanishes linearly as $S \rightarrow 0$, for the class of systems, commonly considered, that have time-reversal symmetry. I soon discovered [4, 5] that this behaviour could be understood by regarding the system under consideration as a member of a family, smoothly labelled by parameters X_1, X_2, \dots . Linear level repulsion has its origin in those members of the family for which two levels coincide, and in order to find such degeneracies it is necessary, in the absence of geometric symmetry (but with time reversal symmetry), to vary two parameters, not one - this is the celebrated 'no-crossing theorem' [6].

These 'generic degeneracies' were becoming interesting in their own right. Previous generations of physicists called them 'accidental' but like traffic accidents they are very unlikely to happen to you (that is, in

any particular system) but become inevitable and predictable in a population (that is in a two-parameter family). So I embarked on a numerical investigation, with my new student Michael Wilkinson, of a two parameter family of quantum systems. We chose triangle billiards, because the space of triangles is two-dimensional (X_1 and X_2 could be two of the angles, for example), and calculated the first thirteen levels as a function of X_1 and X_2 . Many degeneracies appeared, but we faced the question: how can we be sure that two levels really coincide, rather than getting very close and then receding (as with avoided crossings with a single parameter)?

The answer lay in a theorem [7, 8] that I had rediscovered a few years before: around a circuit in $\{X_1, X_2\}$ space that surrounds a degeneracy, the (real) wavefunctions of each of the two degenerating states, when smoothly continued, changes sign. I realized that this is mathematically the same sign that arises when a fermion is rotated through 2π [9, 10]. Detection of this sign change is a sure sign of a degeneracy, and we made use of this in interpreting our numerical results [11].

I talked about the work in a lecture at Georgia Institute of Technology in March 1983. In describing the no-crossing theorem, I mentioned the more general case when there is no symmetry at all - not even under time reversal - for example when there is a magnetic field in the billiard, and the particle moving inside it is charged. Then, three parameters, not two, would be necessary in order to find a degeneracy, and the degeneracies that Wilkinson and I found for unsymmetrical triangles would be destroyed.

At the end of the lecture, the Chairman of the physics department, Professor Ron Fox, asked the innocent question: "When a degeneracy is destroyed by a magnetic field, what happens to the sign change when a wavefunction is continued round the circuit that formerly enclosed it?" I was unable to answer immediately, because I had not thought about this question. During the rest of that visit, and for several weeks after I returned to Bristol, and in many conversations with my colleague Dr John Hannay, I came to appreciate that the generalization of the sign, to systems without time-reversal symmetry, was interesting, and probably deep, because it showed that while a quantum state must be a singlevalued function of its variables (e.g. position) it need not be singlevalued under continuation of parameters in the hamiltonian, and the - geometric - phase factor accompanying such continuation need not be simply a sign.

In May 1983 - after I had written the paper about the geometric phase, but before I sent it for publication - I drove Professor Eric Heller

to my home in Bristol from a meeting in Warwick. During the drive, I talked excitedly about the geometric phase. His response was to draw my attention to recent work by C. Alden Mead [12] about the 'Molecular Aharonov-Bohm effect'. On reading about this, I realized that Mead had clearly anticipated the main idea in the molecular context, but thought that my own paper possessed sufficiently greater generality to be still worth publishing. So, I inserted a reference to Mead, and sent the paper off. It arrived at the Royal Society on 13 June 1983.

Eagerly I awaited the referee's report, but it did not arrive. In August of that year I visited Australia. In Canberra I met Professor Barry Simon, and told him about the phase. At once he appreciated its connection with the Chern class of fibre bundle theory, and on returning to the USA he wrote a Physical Review Letter about the geometrical meaning of what he named 'Berry's phase', and this was published in December 1983 [13]. I had still not received a report from the Royal Society's referee. After I pressed The Society about my paper, they found that the referee (I do not know who it was) had lost his copy of it, and sent another. This time the response was rapid and positive, and my paper was published in February 1984.

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