Quantum Magnetomechanics with Levitating Superconductors

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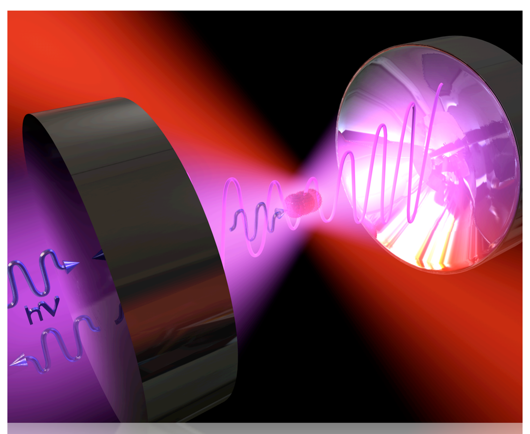
Abstract

We show that by magnetically trapping a superconducting microsphere close to a quantum circuit, it is experimentally feasible to perform ground state cooling and to prepare quantum superpositions of the center-of-mass motion of the microsphere. Due to the absence of clamping losses and time dependent electromagnetic fields, the mechanical motion of micrometer-sized metallic spheres in the Meissner state is predicted to be extremely well isolated from the environment. Hence, we propose to combine the technology of magnetic microtraps and superconducting qubits to bring relatively large objects to the quantum regime.

Quantum Magnetomechanics

Optical Levitation reduces dissipation (Romero-Isart 10)

- Unclamped, optically levitated nanosphere
- Large spatial superpositions possible



Replace lasers with magnetic fields

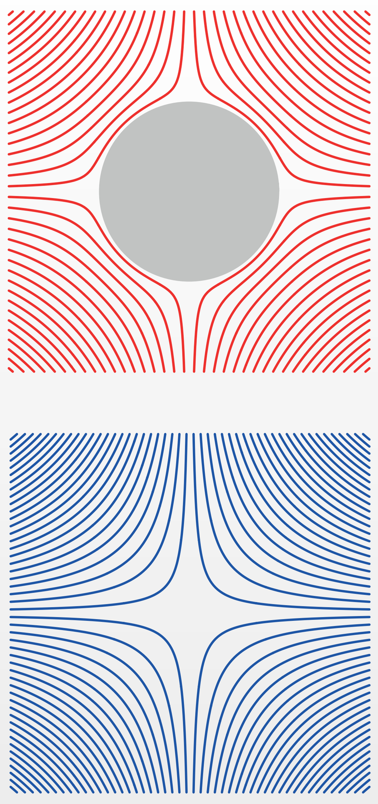
Upper limit on the object size:

- Photon scattering: Position localization
- Photon absorption: Black body radiation

Trapping and Coupling

- Pb sphere of radius R (~μm), mass M (~10^{14} amu), cooled below T_C
- Trapped in a linear magnetostatic field, created by anti-Helmholtz coils
- Sphere expels magnetic field, effective magnetic moment
- Magnetic flux at pickup coil depends on sphere's position

$$\omega_t \approx 1.05\sqrt{\mu_0/\rho}\, I/l^2$$
$$R_{\max} \approx 0.98 B_C/(\omega_t\sqrt{\mu_0\rho})$$
$$\eta = x_0 \Phi'_{\mathrm{ext}}(0)/\Phi_0$$



Sources of Decoherence

- Damping
  - Air friction
  - Hysteresis in the coils
- Decoherence
  - Trap frequency fluctuations
  - Trap center fluctuations
- Advantages
  - Black body radiation negligible (small temperature)
  - No light scattering
  - No clamping losses

$$Q_{\mathrm{air}} = \frac{\omega_{\mathrm{t}}\pi\bar{v}R\rho}{16P} > 10^{10}$$
$$Q_{\mathrm{h}} \propto \frac{\tau^2 l^8}{\bar{x}^4 r^3}\left(\frac{d}{R}\right)^{12}\frac{\hbar\omega_{\mathrm{t}}J_{\mathrm{c}}^2}{\mu_0 I^4} > 10^{10}$$
$$\Gamma_\omega = R^\omega_{0\to 2} = \frac{\pi\omega_{\mathrm{t}}^2}{16}S_\omega(2\omega_{\mathrm{t}}) \sim \mathrm{Hz}$$
$$\Gamma_x = R^x_{0\to 1} = \frac{\pi\omega_{\mathrm{t}}^2}{4}\frac{S_x(\omega_{\mathrm{t}})}{x_{\mathrm{zp}}^2} \sim \mathrm{Hz}$$

Ground State Cooling

Qubit-Sphere Hamiltonian

Hamiltonian for three-junction flux qubit with linear coupling to the sphere:

$$g_0 = \epsilon(0)\eta$$

$$\hat{H}_{\mathrm{MM}} = -\hbar\frac{\epsilon(0)}{2}\hat{\sigma}_z - \hbar\frac{\Delta}{2}\hat{\sigma}_x + \hbar\omega_t \hat{b}^\dagger\hat{b} - g_0\hat{\sigma}_z(\hat{b}^\dagger + \hat{b})$$

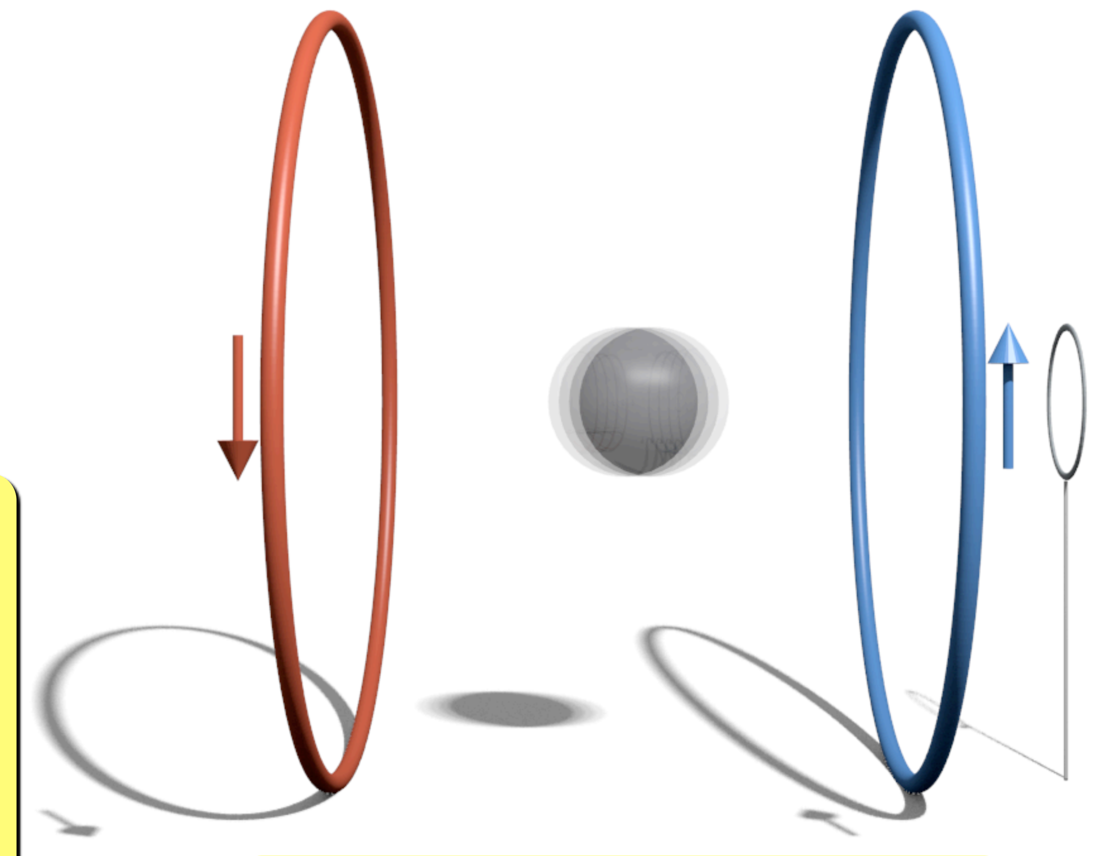
Coupling GHz-MHz is achieved by driving the qubit:

$$\hat{H}_{\mathrm{drive}} = \hbar\Omega\cos(\omega_d t)\hat{\sigma}_z$$

RWA (coupling < qubit resonance), diagonalization, interaction frame, RWA (coupling < trapping):

$$\hat{H}_I = \hbar g(\hat{\sigma}_-\hat{b}^\dagger + \mathrm{H.c})$$

$$g = g_0\cos\alpha\sin\beta$$
$$\tan\alpha = \frac{\Delta}{\epsilon(0)}$$
$$\tan\beta = \frac{\tilde{\Omega}}{\delta\omega}$$
$$\delta\omega = \omega_d - \omega_s$$
$$\tilde{\Omega} = \Omega\sin\alpha$$



$$\partial_x\Phi_{\mathrm{ext}}(0) \approx 2.7\mu_0\frac{I}{l^2}R^3\frac{r^2}{(d^2+r^2)^{3/2}}$$

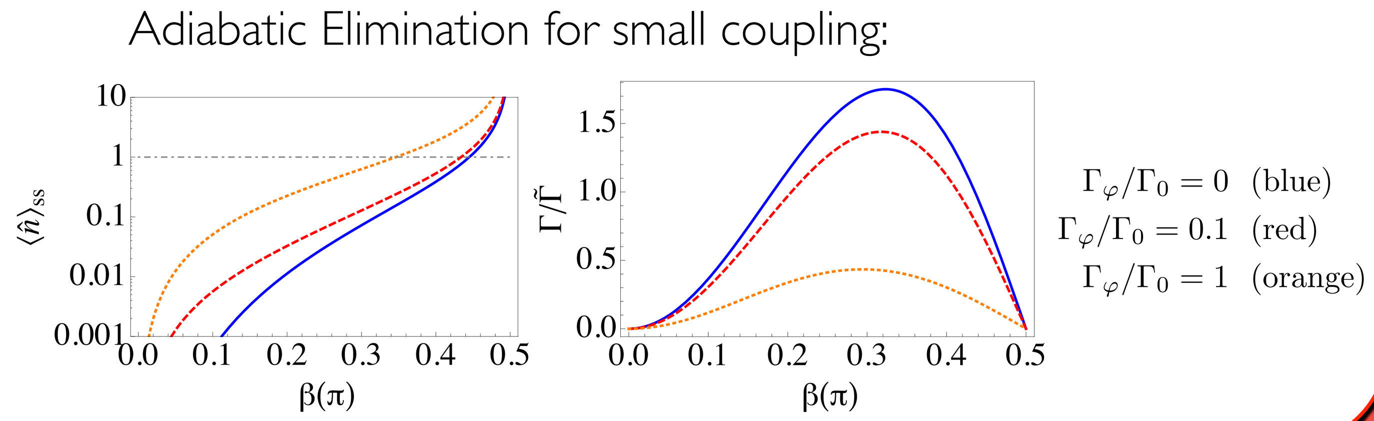
Cooling

Master equation with qubit decay / excitation and dephasing in the interaction frame:

$$\dot{\rho} = \frac{\mathrm{i}}{\hbar}[\hat{H}_I, \rho] + \mathcal{L}_{\Gamma_\uparrow,\Gamma_\downarrow,\Gamma^*_\varphi}[\rho]$$

$$\Gamma^*_\varphi = \Gamma_\varphi\cos^2\beta + \Gamma_0\sin^2(\beta)/2$$
$$\Gamma_{\uparrow/\downarrow} = \Gamma_\varphi\sin^2\beta + \Gamma_0(1\pm\cos\beta)^2/2$$

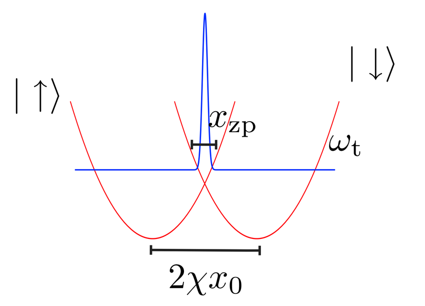
Adiabatic Elimination for small coupling:



Spatial Superposition States

Spin dependent shift:

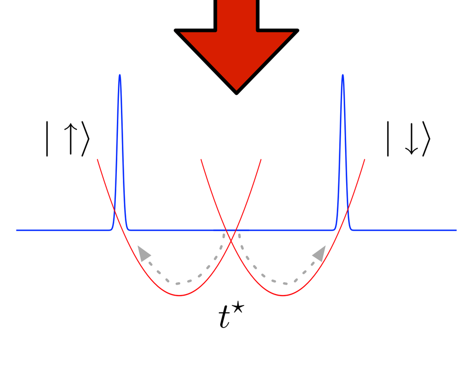
$$\hat{H} = \hbar\frac{\omega_s}{2}\hat{\sigma}_z + \hat{T}(\chi\hat{\sigma}_z)^\dagger\hat{H}_m\hat{T}(\chi\hat{\sigma}_z)$$
$$\hat{H}_m = \frac{\hat{p}^2}{2M} + \frac{M\omega_t^2\hat{x}^2}{2}$$
$$\hat{T}(a) = \exp[-\mathrm{i}\hat{p}a x_0/\hbar]$$
$$\chi = 2g/\omega_t$$



$$\frac{1}{\sqrt{2}}(|\uparrow,0\rangle + |\downarrow,0\rangle)$$

Protocol

- Start in qubit state $|\Psi_0\rangle = |+\rangle$
- Evolve for $t^* = \pi/\omega_t$
- Collapse at t*, Revival after 2 t*



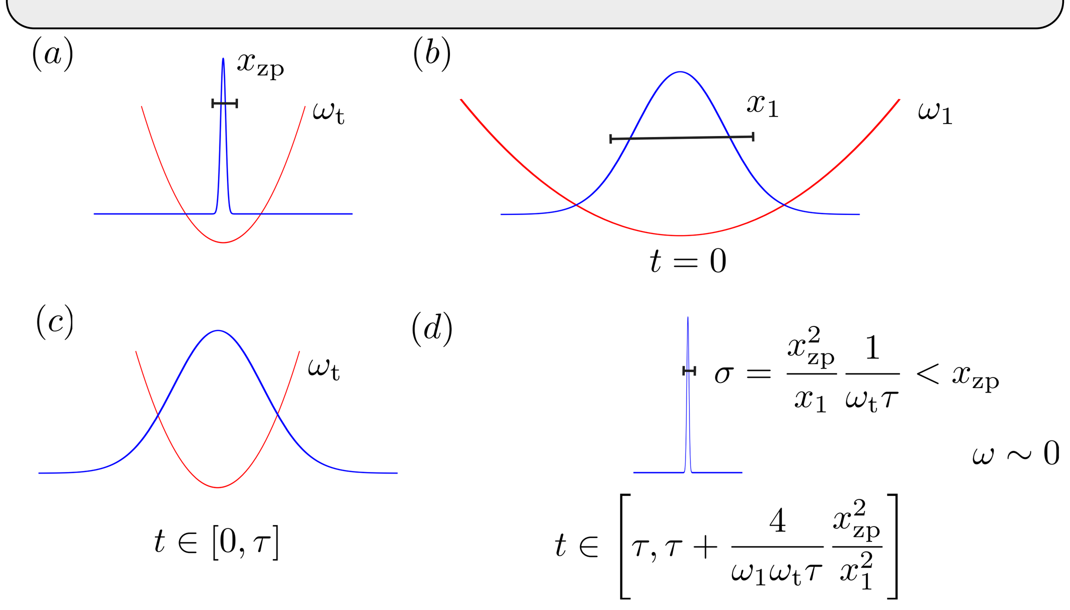
Separation after one step:

$$l_s = 4\chi x_0 = 8x_0 g/\omega_t$$

$$\frac{1}{\sqrt{2}}(\hat{T}(-2\chi)|\uparrow,0\rangle + \hat{T}(2\chi)|\downarrow,0\rangle)$$

Squeezing in x

a) Start in ground state
b) Open trap adiabatically
c) Short closing of the trap yields imaginary phase
d) Open trap, wave function contracts



$$\phi(x) = a\exp(-x^2/4x_1^2 - \mathrm{i}bx^2)$$

Exp. Parameters

- Sphere: 2 μm radius, Pb
- Temperature: 0.1 K
- AHC: 50 μm diameter, 10 A, 16 μm² cross section
- Pickup coil: 28 μm radius, 20 μm distance
- Trap frequency: $\omega_t = 2\pi$ 28 kHz
- Coupling $g_0 = 2\pi$ 1.3 kHz
- Qubit frequency $\omega_s = 2\pi$ 10 GHz, decay rate $\Gamma_0 = 2\pi$ 16 kHz

References

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5. L. Clemente *et al.* (in preparation)